NAG Toolbox for MATLAB

g13ab

1 Purpose

g13ab computes the sample autocorrelation function of a time series. It also computes the sample mean, the sample variance and a statistic which may be used to test the hypothesis that the true autocorrelation function is zero.

2 Syntax

```
[xm, xv, r, stat, ifail] = g13ab(x, nk, 'nx', nx)
```

3 Description

The data consists of n observations x_i, for $i = 1, 2, \ldots, n$ from a time series.

The quantities calculated are

(a) The sample mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}.$$

(b) The sample variance (for $n \geq 2$)

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}.$$

(c) The sample autocorrelation coefficients of lags $k = 1, 2, \ldots, K$, where K is a user-specified maximum lag, and $K < n$, $n > 1$.

The coefficient of lag k is defined as

$$r_k = \frac{\sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}.$$

See page 496 of Box and Jenkins 1976 for further details.

(d) A test statistic defined as

$$\textbf{stat} = n \sum_{k=1}^{K} r_k^2,$$

which can be used to test the hypothesis that the true autocorrelation function is identically zero.

If n is large and K is much smaller than n, **stat** has a χ^2_K distribution under the hypothesis of a zero autocorrelation function. Values of **stat** in the upper tail of the distribution provide evidence against the hypothesis; g01ec can be used to compute the tail probability.

Section 8.2.2 of Box and Jenkins 1976 provides further details of the use of **stat**.

4 References

Box G E P and Jenkins G M 1976 *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **x(nx) – double array**

The time series, x_i, for $i = 1, 2, \ldots, n$.

2: **nk – int32 scalar**

K, the number of lags for which the autocorrelations are required. The lags range from 1 to K and do not include zero.

Constraint: $0 < \mathbf{nk} < \mathbf{nx}$.

5.2 Optional Input Parameters

1: **nx – int32 scalar**

Default: The dimension of the array **x**.

n, the number of values in the time series.

Constraint: $\mathbf{nx} > 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **xm – double scalar**

The sample mean of the input time series.

2: **xv – double scalar**

The sample variance of the input time series.

3: **r(nk) – double array**

The sample autocorrelation coefficient relating to lag k, for $k = 1, 2, \ldots, K$.

4: **stat – double scalar**

The statistic used to test the hypothesis that the true autocorrelation function of the time series is identically zero.

5: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{nx} \leq \mathbf{nk}$,
or $\mathbf{nx} \leq 1$,
or $\mathbf{nk} \leq 0$.

ifail $= 2$

On entry, all values of **x** are practically identical, giving zero variance. In this case **r** and **stat** are undefined on exit.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by g13ab is approximately proportional to $\mathbf{nx} \times \mathbf{nk}$.

If the input series for g13ab was generated by differencing using g13aa, ensure that only the differenced values are input to g13ab, and not the reconstituting information.

9 Example

```
x = [5;
     11;
     16;
     23;
     36;
     58;
     29;
     20;
     10;
     8;
     3;
     0;
     0;
     2;
     11;
     27;
     47;
     63;
     60;
     39;
     28;
     26;
     22;
     11;
     21;
     40;
     78;
     122;
     103;
     73;
     47;
     35;
     11;
     5;
     16;
     34;
```

```
     70;
     81;
     111;
     101;
     73;
     40;
     20;
     16;
     5;
     11;
     22;
     40;
     60;
     80.900000000000001];
nk = int32(10);
[xm, xv, r, stat, ifail] = g13ab(x, nk)
```

```
xm =
   37.4180
xv =
   1.0020e+03
r =
    0.8004
    0.4355
    0.0328
   -0.2835
   -0.4505
   -0.4242
   -0.2419
    0.0550
    0.3783
    0.5857
stat =
   92.1231
ifail =
           0
```